$$pp^* = \left(\sum^N a_i z^i\right)\left(\sum^N a_i^* \overset{z^*}{\widetilde{z^{-1}}}{}^{i}\right) = \sum_i^N a_i \sum_k^N a_k^* z^i z^{-k} = \sum_i^N a_i \left(\sum_{k=i}^N a_{k}^* z^{i-k} + \sum_{k=0}^{i} a_k^* z^{i-k}\right)$$

$$a_i^* = a_i$$

$$= \cancel{\sum_i^N \sum_{k=i}^{N-i} a_i a_{k+i} z^k} = \sum_i^N a_i \left(\sum_{k=0}^{N-i} a_{k+i}^* z^k + \underbrace{\sum_{k=-i}^{0} a_{k+i} z^k}_{\sum_{k=0}^{i} a_{-k+i} z^{-k}}\right)$$

$$b_i = \sum_k^{N-i} a_k a_{i-k+N}^*$$

$$pp^* = \frac{1}{2}\sum_{-N}^{N} \underbrace{b_i (z+z^{-1})}_{\text{DCT}}$$

$$= \sum$$

$$pp^* = \boxed{b_0 + \sum_{i=1}^N b_i \frac{1}{2}(z^{i} + z^{-i})}$$ easy to fit (pure real)

reinterpret

$$\frac{z^N pp^*}{z^N} = \frac{1}{z^N}\left[b_0 z^N + \sum \frac{b_i}{2}\left(z^{N+i} + z^{N-i}\right)\right]$$

Palindromic polynomial
(self-reciprocal)

$\boxed{\text{all roots } \alpha \text{ also have inverses } \alpha^{-1}}$